

$$1. a) S.A = 4\pi r^2$$

$$S.A = 12.568 r^2$$

$$b) V = \frac{4}{3}\pi r^3$$

$$3V = 4\pi r^3$$

$$4\pi r^3 = 3V$$

$$r^3 = \frac{3V}{4\pi}$$

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

or

$$r = 2.325 \sqrt[3]{3V}$$

$$c) S.A(r(V))$$

$$12.568 r^2 (2.325 \sqrt[3]{3V} \times 12.568 r^2)$$

$$12.568 r^2 (9.7402 r^3 \sqrt[3]{V})$$

$$122.4148 r^5 \sqrt[3]{V}$$

$$d) V = \frac{4}{3}\pi r^3$$

$$0.75 = \frac{4}{3} \times 3.142 \times r^3$$

$$r^3 = \frac{0.75 \times 3}{4 \times 3.142}$$

$$r^3 = 0.17903$$

$$r = 0.5636$$

$$S.A = 4\pi r^2$$

$$= 4 \times 3.142 \times 0.5636^2$$

$$= 1.268 m^2$$

$$2. a) f \circ g(x)$$

$$f(x) = x+2$$

$$g(x) = x^2 - 4$$

$$f \circ g(x) = x^2 - 4 + 2$$

$$f \circ g(x) = x^2 - 2$$

$$b) g \circ f(x)$$

$$(x+2)^2 - 4$$

$$(x+2)(x+2) - 4$$

$$x^2 + 4x + 4 - 4$$

$$x^2 + 4x$$

$$g \circ f(x) = x^2 + 4x$$

or

$$x(x+4)$$

$$c) (h+g)(-1)$$

$$h(x) = \frac{1}{\sqrt{x+1}} \quad g(x) = x^2 - 4$$

$$(h+g)(-1)$$

$$\left(\frac{1}{\sqrt{x+1}} + x^2 - 4 \right) (-1)$$

$$= \frac{1}{\sqrt{(-1)+1}} + (-1)^2 - 4$$

$$= \frac{1}{0} + 1 - 4$$

$$= \alpha$$

Exponential and Logarithmic Functions

5. a) $5^{x+1} = 6^{x+1}$

$$\log 5^{x+1} = \log 6^{x+1}$$

$$(x+1) \log 5 = (x+1) \log 6$$

$$\frac{x+1}{x+1} = \frac{\log 6}{\log 5}$$

$$\frac{x+1}{x+1} = 1.113$$

$$1.113x + 1.113 = x + 1$$

$$1.113x - x = 1 - 1.113$$

$$0.113x = -0.113$$

$$x = \frac{-0.113}{0.113}$$

$$x = -1$$

b) $3^{5x} \cdot 9^{x^2} = 27$

$$3^{5x} \times 3^{2x^2} = 3^3$$

$$5x + 2x^2 = 3$$

$$2x^2 + 5x - 3 = 0$$

$$a+b = 5 \quad (6, -1)$$

$$a \times b = -6$$

$$2x^2 + 6x - x - 3 = 0$$

$$2x(x+3) - 1(x+3) = 0$$

$$(2x-1)(x+3) = 0$$

$$x = -3 \text{ or } \frac{1}{2}$$

6a) $t = \log \left(\frac{T-22}{25} \right) \div \log 0.75$

$$T = 35^\circ\text{C}$$

$$t = \log \left(\frac{35-22}{25} \right) \div \log 0.75$$

$$t = \log 0.52 \div \log 0.75$$

$$t = 2.273$$

$$\approx 2 \text{ minutes}$$

b) Initial temperature $t = 0$

$$0 = \log \left(\frac{T-22}{25} \right) \div \log 0.75$$

$$\log 1 = \log \left(\frac{T-22}{16.5} \right)$$

$$\frac{T-22}{16.5} = 1$$

$$16.5$$

$$T - 22 = 16.5$$

$$T = 16.5 + 22$$

$$T = 38.5^\circ\text{C}$$

$$1. a) \log_3 \frac{1}{9}$$

$$\log_3 9^{-1} = \log_3 3^{-2}$$

$$= -2 \log_3 3$$

$$\log_3 3 = 1$$

$$= -2 \times 1$$

$$= -2$$

$$3. \log a + \log b = \log ab$$

$$a \log b = \log(b^a)$$

$$\log a - \log b = \log\left(\frac{a}{b}\right)$$

$$4 a) \log_4 (x+2) + \log_4 (x-1) = 1$$

$$\log_4 (x+2) + \log_4 (x-1) = \log_4 4$$

$$\log_4 (x+2)(x-1) = \log_4 4$$

$$(x+2)(x-1) = 4$$

$$x^2 + x - 2 = 4$$

$$x^2 + x - 2 - 4 = 0$$

$$x^2 + x - 6 = 0$$

$$a+b=1 \quad (-2, 3)$$

$$a \times b = -6$$

$$x^2 - 2x + 3x - 6 = 0$$

$$x(x-2) + 3(x-2) = 0$$

$$(x+3)(x-2) = 0$$

$$x = 2 \text{ or } -3$$

$$c) \log 2 + \log 5 - \log 25$$

$$\log(2 \times 5) - \log 5^2$$

$$\log 10 - 2 \log 5$$

$$1 - 2$$

$$= -1$$

$$2. \log_4 x^2 + 3 \log_3 \frac{1}{3} y - \log_4 x$$

$$\log_4 \left(x^2 \times \frac{1}{27} y^3 \times \frac{1}{x} \right)$$

$$\log_4 \left(\frac{x^2 y^3}{27} \right)$$

$$= \log_4 \left(\frac{x^2 y^3}{27} \right)$$

6.

$$b) \log_3(8x-2) + \log_3(x-1) = 2.$$

$$\log_3(8x-2) + \log_3(x-1) = \log_3 9$$

$$\log_3(8x-2)(x-1) = \log_3 9$$

$$(8x-2)(x-1) = 9$$

$$8x^2 - 10x + 2 = 9$$

$$8x^2 - 10x + 2 - 9 = 0$$

$$8x^2 - 10x - 7 = 0.$$

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4 \times 8 \times -7}}{2 \times 8}$$

$$x = \frac{10 \pm 18}{16}$$

$$x = \frac{10+18}{16} \text{ or } \frac{10-18}{16}$$

$$= 1.75 \text{ or } -0.5.$$

Trigonometric functions

1) $\cos^4 x - \sin^4 x = \cos 2x$

$$x^2 - y^2 = (x+y)(x-y)$$

$$\cos^4 x - \sin^4 x =$$

$$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$$

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

2) $\cos^4 x - \sin^4 x = \cos 2x$

2. a) $\frac{2}{\sin x} + 10 = 6$

$$\frac{2}{\sin x} = -4$$

$$\sin x = \frac{2}{-4}$$

$$\sin x = -0.5$$

$$x = \sin^{-1} 0.5$$

$$x = \frac{\pi}{6}$$

where x is negative.

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$= 210^\circ, 330^\circ$$

b) $3 + 10 \sec x - 1 = -18$

$$10 \sec x = -18 - 3 + 1$$

$$10 \sec x = -20$$

$$\sec x = -2$$

$$x = \sec^{-1} -2$$

$$\sec x = \frac{1}{\cos x}$$

$$\frac{1}{\cos x} = -2$$

$$\cos x = -0.5$$

$$x = \cos^{-1} 0.5$$

$$x = 60^\circ = \frac{\pi}{3}$$

where x is negative

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$= 120^\circ, 240^\circ$$

3 a) $2 \sin x - 1 = 0$ $\cos x = 0$

$$2 \sin x - 1 = 0$$

$$\sin x = 0.5$$

$$x = \sin^{-1} 0.5$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$= 30^\circ, 150^\circ$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$= 30^\circ, 150^\circ$$

$$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$$

$$= 30^\circ, 90^\circ, 150^\circ, 270^\circ$$

$$b) \cos^2 x = 2 + \cos x$$

$$\cos^2 x - \cos x - 2 = 0$$

$$a+b = -1$$

$$a \times b = -2$$

$$\cos^2 x - 2\cos x + (\cos x - 2) = 0$$

$$\cos x (\cos x - 2) + 1(\cos x - 2) = 0$$

$$(\cos x + 1)(\cos x - 2) = 0$$

$$\cos x + 1 = 0 \quad \cos x - 2 = 0$$

$$\cos x = -1 \quad \cos x = 2$$

$$x = \cos^{-1} 1 \quad x = \cos^{-1} 2$$

$$x = 180^\circ = \pi \quad x = 0$$

$$\therefore x = 0, 180^\circ$$

$$= \pi$$

$$A. d(t) = 4 + 2 \sin\left(\frac{\pi}{6}t\right)$$

$$d = 3$$

$$3(t) = 4 + 2 \sin\frac{\pi}{6}t$$

$$3(t) - 2 \sin\frac{\pi}{6}t = 4$$

$$3t - 2 \times 0.5t = 4$$

$$3t - t = 4$$

$$2t = 4$$

$$t = 2 \text{ hours}$$

$$5. 2 \sin x \sec x - 2\sqrt{3} \sin x = 0$$

$$\sin x \sec x - \sqrt{3} \sin x = 0$$

$$\sin x \sec x = \sqrt{3} \sin x$$

$$\sec x = \sqrt{3}$$

$$\frac{1}{\cos x} = \sqrt{3}$$

$$\cos x$$

$$\cos x = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}$$

$$x = \cos^{-1} \frac{1}{\sqrt{3}}$$

$$\sqrt{3}$$

$$x = 0.499 \text{ } 54.7353^\circ$$

$$x = 54.7353^\circ, 305.2647^\circ$$

$$x = 54.74^\circ, 305.26^\circ$$

$$3) f(x) = 5x + 18$$

$$g(x) = 2x^2$$

$$2x^2 = 5x + 18$$

$$2x^2 - 5x - 18 = 0$$

$$a+b = -5 \quad -9, 4$$

$$a \times b = -36$$

$$2x^2 - 9x + 4x + 18 = 0$$

$$x(2x - 9) + 2(2x - 9) = 0$$

$$(x+2)(2x-9) = 0$$

$$x+2=0 \quad 2x-9=0$$

$$x = -2 \quad 2x = 9$$

$$x = 4.5$$

$$x = -2 \text{ or } 4.5$$

$\therefore$ Number of product = $4 \cdot 5$

$$4) f(x) = 4 \tan 5x$$

$$g(x) = 3 \cos 5x$$

$$(f \times g)(x)$$

$$= 4 \tan 5(3 \cos 5x)$$

$$(f \times g)(x) = 4 \tan(5 \cdot 3 \cos(5x))$$

$$b) f(x) = \sqrt{5}x^2 \text{ and } g(x) = 5\sqrt{5}x^2$$

$$(f \times g)(x)$$

$$= \sqrt{5(5\sqrt{5}x^2)^2}$$

$$= \sqrt{5(125x^2)}$$

$$= \sqrt{625x^2}$$

$$(f \times g)(x) = 25\sqrt{x^2}$$